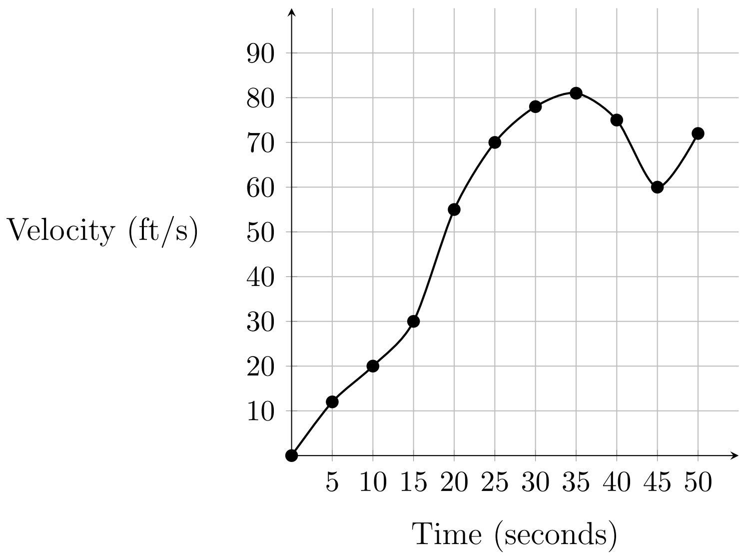


t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

- The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

(a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

(b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \leq t \leq 50$.

(c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.

(d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

Solutions

(a) Acceleration is positive on $(0, 35)$ and $(45, 50)$ because the velocity $v(t)$ is increasing on $[0, 35]$ and $[45, 50]$

(b) Avg. Acc. $= \dfrac{v(50) - v(0)}{50 - 0} = \dfrac{72 - 0}{50} = \dfrac{72}{50}$
or 1.44 ft/sec^2

(c) Difference quotient; e.g.
$\dfrac{v(45) - v(40)}{5} = \dfrac{60 - 75}{5} = -3$ ft/sec^2 or
$\dfrac{v(40) - v(35)}{5} = \dfrac{75 - 81}{5} = -\dfrac{6}{5}$ ft/sec^2 or

$\dfrac{v(45)-v(35)}{10} = \dfrac{60-81}{10} = -\dfrac{21}{10}$ ft/sec^2

–or–

Slope of tangent line, e.g.

through $(35, 90)$ and $(40, 75)$: $\dfrac{90-75}{35-40} = -3$ ft/sec^2

(d) $\displaystyle\int_0^{50} v(t)\,dt$

$\approx 10[v(5)+v(15)+v(25)+v(35)+v(45)]$

$= 10(12+30+70+81+60)$

$= 2530$ feet

This integral is the total distance traveled in feet over the time 0 to 50 seconds.

- A particle moves along the y axis with velocity given by $v(t) = t\sin(t^2)$ for $t \geq 0$.

(a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

(b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?

(c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

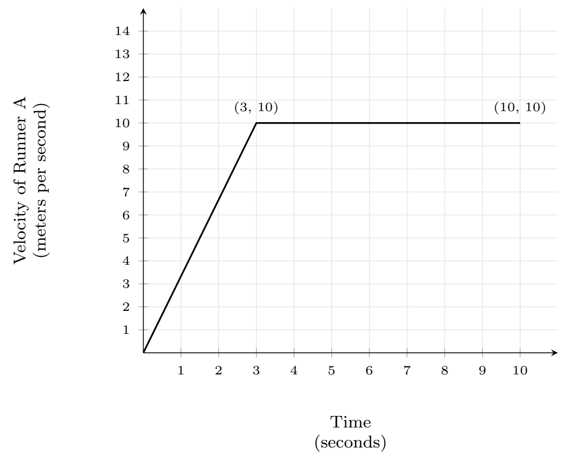
Solutions

(a) $v(1.5) = 1.5\sin(1.5^2) = 1.167$
Up, because $v(1.5) > 0$

(b) $a(t) = v'(t) = \sin t^2 + 2t^2 \cos t^2$
$a(1.5) = v'(1.5) = -2.048 \text{ or } -2.049$
No; v is decreasing at 1.5 because $v'(1.5) < 0$

(c) $$y(t) = \int v(t)\,dt$$
$$= \int t\sin t^2\,dt = -\frac{\cos t^2}{2} + C$$
$y(0) = 3 = -\frac{1}{2} + C \implies C = \frac{7}{2}$
$y(t) = -\frac{1}{2}\cos t^2 + \frac{7}{2}$
$y(2) = -\frac{1}{2}\cos 4 + \frac{7}{2} = 3.826 \text{ or } 3.827$

(d) $$\text{distance} = \int_0^2 |v(t)|\,dt = 1.173$$
or
$v(t) = t\sin t^2 = 0$
$t = 0 \text{ or } t = \sqrt{\pi} \approx 1.772$
$y(0) = 3;\ y(\sqrt{\pi}) = 4;\ y(2) = 3.826 \text{ or } 3.827$
$[y(\sqrt{\pi}) - y(0)] + [y(\sqrt{\pi}) - y(2)]$
$= 1.173 \text{ or } 1.174$



Two runners, A and B, run on a straight racetrack for $0 \leq t \leq 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.

(a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

(b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \leq t \leq 10$ seconds. Indicate units of measure.

Solutions

(a) Runner A: velocity $= \frac{10}{3} \cdot 2 = \frac{20}{3}$
$= 6.666$ or 6.667 meters/sec

Runner B: $v(2) = \frac{48}{7} = 6.857$ meters/sec

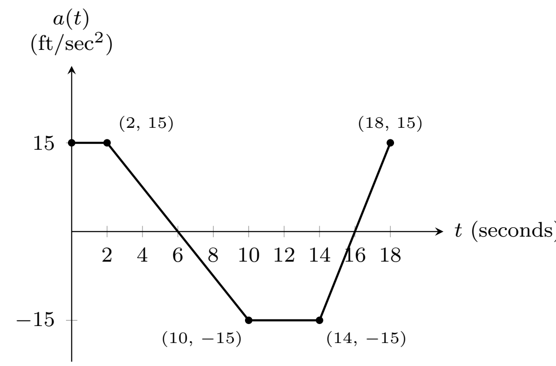
(b) Runner A: acceleration $= \frac{10}{3} = 3.333$ meters/sec^2

Runner B: $a(2) = v'(2) = \left.\frac{72}{(2t+3)^2}\right|_{t=2}$
$= \frac{72}{49} = 1.469$ meters/sec^2

(c) Runner A: distance $= \frac{1}{2}(3)(10) + 7(10) = 85$ meters

Runner B: distance $= \int_0^{10} \frac{24t}{2t+3}\, dt = 83.336$ meters

(units) meters/sec in part (a),
meters/sec^2 in part (b), and
meters in part (c), or equivalent.



A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \leq t \leq 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \leq t \leq 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \leq t \leq 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

(d) At what times in the interval $0 \leq t \leq 18$, if any, is the car's velocity equal to zero? Justify your answer.

Solutions

(a) Since $v'(2) = a(2)$ and $a(2) = 15 > 0$, the velocity is increasing at $t = 2$.

(b) At time $t = 12$ because

$$v(12) - v(0) = \int_0^{12} a(t)\,dt = 0.$$

(c) The absolute maximum velocity is 115 ft/sec at $t = 6$.
The absolute maximum must occur at $t = 6$ or at an endpoint.

$$\begin{aligned} v(6) &= 55 + \int_0^6 a(t)\,dt \\ &= 55 + 2(15) + \frac{1}{2}(4)(15) = 115 > v(0) \end{aligned}$$

$$\int_6^{18} a(t)\,dt < 0 \text{ so } v(18) < v(6)$$

(d) The car's velocity is never equal to 0. The absolute minimum occurs at $t = 16$ where

$$v(16) = 115 + \int_6^{16} a(t)\,dt = 115 - 105 = 10 > 0.$$

Problems adapted from the College Board Question Bank and released practice tests.