Interpreting the Initial Value

• A population of a certain species of bird is modeled by the function $P(t) = 1,500(1.08)^t$, where P is the population and t is the number of years after 2010. What does the value 1,500 represent in this context?

(A) The population of the bird species in 2010

(B) The population of the bird species in 2020

(C) The population of the bird species increases by 1500 each year

(D) The population of the bird species decreases by 1500 each year

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Solution:
In the standard exponential form $f(x) = a(b)^x$, a represents the initial value (when $x = 0$). Here, $t = 0$ corresponds to the year 2010. Thus, 1,500 is the initial population of the bird species in the year 2010.

Answer: (A)

• Which of the following functions represents exponential decay?

(A) $f(x) = 3(1.5)^x$

(B) $f(x) = 0.4(0.8)^x$

(C) $f(x) = 2(1.05)^x$

(D) $f(x) = 5(2)^x$

Solution:
An exponential function $f(x) = a \cdot b^x$ represents decay when the base satisfies $0 < b < 1$.

- (A) base $= 1.5 > 1$ growth
- (B) base $= 0.8$, and $0 < 0.8 < 1$ decay ✓
- (C) base $= 1.05 > 1$ growth
- (D) base $= 2 > 1$ growth

Answer: (B)

- The number of bacteria in a culture is modeled by

$$N(t) = 500 \cdot (1.08)^t$$

where t is time in hours. What does the value 1.08 represent?

(A) The initial number of bacteria

(B) The number of bacteria added each hour

(C) The bacteria count doubles every 1.08 hours

(D) The bacteria population grows by 8% each hour

Solution:

In the general model $N(t) = N_0 \cdot (1 + r)^t$:

- $N_0 = 500$ is the initial number of bacteria at $t = 0$.
- $1 + r = 1.08$, so $r = 0.08 = 8\%$ is the hourly growth rate.

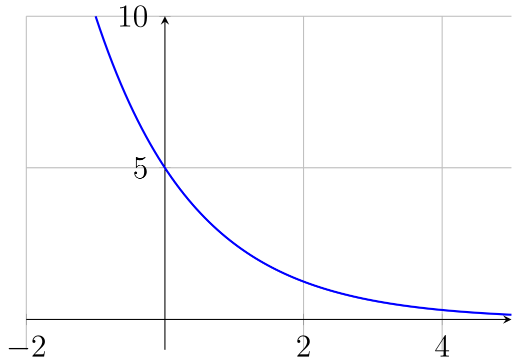
The base 1.08 means the population is multiplied by 1.08 — i.e., grows by 8% — each hour.

Answer: (D)

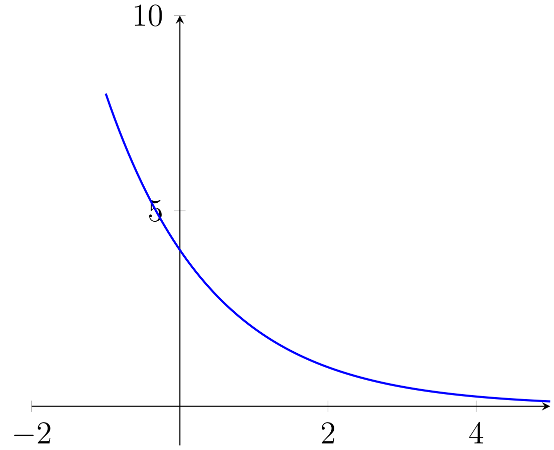
Exponential Decay and Graphs

- Which of the following could be the graph of $y = 5(0.5)^x$ in the xy-plane?

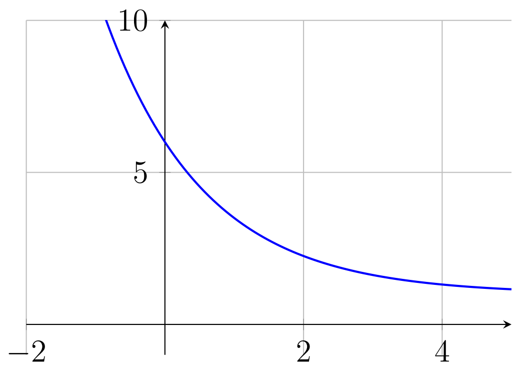
A)



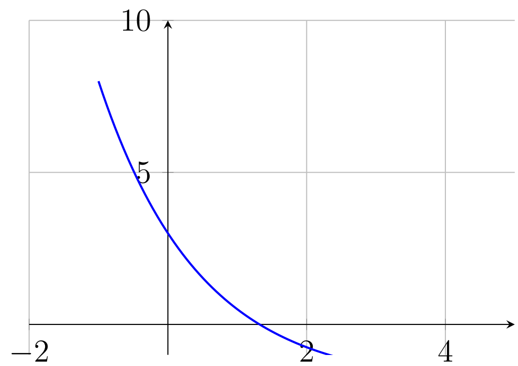
B)



C)



D)



Solution:
Analyze the equation $y = 5(0.5)^x$:

1. Initial Value: When $x = 0$, $y = 5(0.5)^0 = 5$. The graph must cross the y-axis at $(0, 5)$.
2. Behavior: Since the base (0.5) is between 0 and 1, the function represents **decay**. As x increases, y must decrease.

The graph shown above correctly starts at 5 and curves downward toward the x-axis.

Answer: A

Modeling from a Table

- The table below shows the population of a colony of bacteria at different times.

Time (t, hours)	Population (P)
0	400
1	1,200
2	3,600

Which equation best models this data?

(A) $P(t) = 400(1.3)^t$

(B) $P(t) = 400(0.3)^t$

(C) $P(t) = 400(3)^t$

(D) $P(t) = 400(3.2)^t$

Solution:
Check the ratio between consecutive P values: $1,200/400 = 3$ and $3,600/1,200 = 3$. This is an exponential function with an initial value $a = 400$ and a growth factor $b = 3$. The equation is $P(t) = 400(3)^t$.

Answer: C

Percent Increase / Decrease

• An investment account earns interest at a rate of 3.5% per year, compounded annually. If the initial deposit was $2,000, which of the following functions models the value $V(t)$, in dollars, of the account after t years?
A) $V(t) = 2,000(0.035)^t$
B) $V(t) = 2,000(0.965)^t$
C) $V(t) = 2,000(1.035)^t$
D) $V(t) = 2,000(1.35)^t$

Solution:
Growth rate is 3.5%. The growth factor b is calculated as $1 + r$.

$$r = 0.035 \implies b = 1 + 0.035 = 1.035$$

The formula is $V(t) = 2000(1.035)^t$.
Answer: C

• The value of a car depreciates by 12% each year. If the car was purchased for $25,000, what is the value of the car after 5 years, to the nearest dollar?

Solution:
A decrease of 12% means the car retains $100\% - 12\% = 88\%$ of its value each year. Growth factor $b = 0.88$.

$$V(5) = 25,000(0.88)^5$$
$$V(5) = 25,000(0.52773) \approx 13,193$$

Answer: $13,193

• A car is purchased for $24,000 and loses 15% of its value each year. What is the value of the car after 5 years, to the nearest dollar?

(A) $ 10,623

(B) $ 12,000

(C) $ 20,400

(D) $ 6,300

Solution:
Each year the car retains $1 - 0.15 = 0.85$ of its value: $V(t) = 24{,}000 \cdot (0.85)^t$

$$V(5) = 24{,}000 \cdot (0.85)^5 = 24{,}000 \cdot 0.4437 \approx \$10{,}623$$

Answer: (A)

Doubling Time and Half-life decay

• A sample of a substance doubles in mass every 3 hours. If the initial mass is 10 grams, which equation gives the mass M, in grams, after t hours?

A) $M = 10(2)^t$
B) $M = 10(2)^{3t}$
C) $M = 10(2)^{t/3}$
D) $M = 10(3)^{t/2}$

Solution:
For doubling time d, the formula is $M = a(2)^{t/d}$.
Here, $a = 10$ and $d = 3$.
So, $M = 10(2)^{t/3}$.

Check:
At $t = 3$, $M = 10(2)^{3/3} = 20$ (doubled), which is correct.

Answer: C

• A radioactive substance has a half-life of 10 years. If the initial amount is 200 grams, which function models the amount $A(t)$ remaining after t years?

(A) $A(t) = 200(2)^{t/10}$

(B) $A(t) = 200\left(\frac{1}{2}\right)^{10t}$

(C) $A(t) = 200\left(\frac{1}{2}\right)^{t/10}$

(D) $A(t) = 200 - 10t$

Solution:

Every 10 years the amount is multiplied by $\frac{1}{2}$. After t years, $\frac{t}{10}$ half-lives have elapsed:

$$A(t) = 200\left(\frac{1}{2}\right)^{t/10}$$

Check: $A(10) = 200\left(\frac{1}{2}\right)^1 = 100$ ✓ $A(20) = 200\left(\frac{1}{2}\right)^2 = 50$
Answer: (C)

Growth Factor Interpretation

- In the function $f(x) = 200(1.45)^x$, what is the percent increase described by the function?

(A) 0.45

(B) 1.45

(C) 145

(D) 200

Solution:
The growth factor b is 1.45. To find the rate r, solve $1 + r = 1.45$.

$$r = 0.45$$

To convert the decimal to a percentage, multiply by 100: 45%.

Answer: A

- An exponential function of the form $f(x) = a \cdot b^x$ passes through (2, 12) and (4, 48). What is the value of b?

(A) $\sqrt{2}$

(B) 2

(C) 3

(D) 4

Solution:
Divide the second equation by the first to eliminate a:

$$\frac{f(4)}{f(2)} = \frac{a \cdot b^4}{a \cdot b^2} = b^2 = \frac{48}{12} = 4$$

$$b^2 = 4 \Rightarrow b = 2 \quad (b > 0)$$

Then $a \cdot 4 = 12 \Rightarrow a = 3$, so $f(x) = 3 \cdot 2^x$.
Verify: $f(2) = 3 \cdot 4 = 12$ ✓ $f(4) = 3 \cdot 16 = 48$

Answer: (B)

Linear vs. Exponential Growth

• Function L is a linear function where $L(0) = 10$ and $L(1) = 12$. Function E is an exponential function where $E(0) = 10$ and $E(1) = 12$. What is $E(2) - L(2)$?

Solution:
Linear L: Slope $m = 12 - 10 = 2$. $L(x) = 2x + 10$. $L(2) = 2(2) + 10 = 14$.

Exponential E: Growth factor $b = 12/10 = 1.2$. $E(x) = 10(1.2)^x$.

$E(2) = 10(1.2)^2 = 10(1.44) = 14.4$.

Difference: $14.4 - 14 = 0.4$.

Answer: 0.4

• The table below shows values of two functions f and g.

x	$f(x)$	$g(x)$
0	2	1
1	4	3
2	8	5
3	16	7
4	32	9

Which statement best describes f and g?

(A) Both are linear

(B) f is linear; g is exponential

(C) f is exponential; g is linear

(D) Both are exponential

Solution:

For f: each output is multiplied by 2 as x increases by 1.

Constant ratio $= 2 \Rightarrow$ **exponential**: $f(x) = 2^{x+1}$.

For g: each output increases by 2 as x increases by 1.

Constant difference $= 2 \Rightarrow$ **linear**: $g(x) = 2x + 1$.

Answer: (C)

Changing the Time Unit

• The population of a city is growing at a rate of 2% per year. The population can be modeled by $P = a(1.02)^t$, where t is the number of years. Which of the following equations models the population growth monthly?

A) $P = a(1.02)^{12t}$
B) $P = a(1.02^{1/12})^{12t}$
C) $P = a(1.02/12)^{12t}$
D) $P = a(1.02 - 12)^t$

Solution:

We want to keep the total annual growth the same. If m is the number of months ($m = 12t$), we need a new base B such that $B^{12} = 1.02$. $B = 1.02^{1/12}$.

Substituting this into the equation: $P = a(1.02^{1/12})^{12t}$. This ensures that when $t = 1$ (one year), the exponent is 12 (months), and the growth is still 1.02.

Answer: B

Comparison of Two Models

- Two bank accounts both start with $\$1,000$. Account A earns 5% simple interest annually. Account B earns 5% interest compounded annually. After 2 years, how much more money is in Account B than Account A?

Solution:

Account A (Simple):

$Interest = P \times r \times t = 1000 \times 0.05 \times 2 = 100.$

$\text{Total} = 1000 + 100 = 1100.$

Account B (Compound): $V = 1000(1.05)^2 = 1000(1.1025) = 1102.5.$

Difference: $1102.5 - 1100 = 2.5.$

Answer: \$2.50

Compound Interest

- $$A = P\left(1+\frac{r}{n}\right)^{nt}$$

An account pays 4% annual interest compounded **quarterly**. If $5,000 is deposited, what is the amount after 3 years?

(A) $ 5,000.00

(B) $ 5,624.32

(C) $ 5,737.45

(D) $ 6,200.00

Solution:

Here $P = 5000$, $r = 0.04$, $n = 4$ (quarterly), $t = 3$:

$$A = 5000\left(1+\frac{0.04}{4}\right)^{4\cdot 3} = 5000(1.01)^{12}$$

$$(1.01)^{12} \approx 1.12683 \;\Rightarrow\; A \approx \$5{,}634.13$$

Answer: (B)

Rewriting Exponential Expressions

Which of the following is equivalent to $f(x) = 8 \cdot 2^{3x}$?

(A) $f(x) = 8^x$

(B) $f(x) = 2^{3x+3}$

(C) $f(x) = 16^x$

(D) $f(x) = 2 \cdot 8^x$

Solution:
Write $8 = 2^3$ so that all terms share the same base:

$$f(x) = 2^3 \cdot 2^{3x} = 2^{3x+3}$$

Verify (D): $2 \cdot 8^x = 2^1 \cdot (2^3)^x = 2^{3x+1} \neq 2^{3x+3}$.

Verify (B): $2^{3x+3}\big|_{x=0} = 2^3 = 8$, and $8 \cdot 2^0 = 8$.

Answer: (B)

Problems adapted from the College Board SAT Question Bank and released SAT practice tests.