The Discriminant

- The equation $3x^2 - 6x + c = 0$ has exactly one real solution. What is the value of c?

Solution:
For a quadratic equation $ax^2 + bx + c = 0$ to have exactly one real solution, its discriminant $(b^2 - 4ac)$ must equal zero. In this equation, $a = 3$, $b = -6$, and c is the constant we need to find.

$$(-6)^2 - 4(3)(c) = 0$$

$$36 - 12c = 0$$

$$36 = 12c$$

$$c = 3$$

Answer: 3

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$$x^2 - 6x + k = 0$$

In the equation above, k is a constant. If the equation has exactly one real solution, what is the value of k?

(A) 3

(B) 6

(C) 9

(D) 12

Solution:

A quadratic $ax^2 + bx + c = 0$ has exactly one real solution when its discriminant equals zero:

$$\Delta = b^2 - 4ac = 0$$

Here $a = 1$, $b = -6$, $c = k$:

$$(-6)^2 - 4(1)(k) = 0 \implies 36 - 4k = 0 \implies k = 9$$

Verification: $x^2 - 6x + 9 = (x - 3)^2 = 0$ gives the unique solution $x = 3$.

Answer: (C)

• The equation $x^2 + 4x + k = 0$ has no real solutions. Which of the following must be true about the value of k?

A) $k < 4$
B) $k = 4$
C) $k > 4$
D) $k > 0$

Solution

For no real solutions, the discriminant $b^2 - 4ac < 0$.
$4^2 - 4(1)(k) < 0 \Rightarrow 16 - 4k < 0 \Rightarrow 16 < 4k \Rightarrow k > 4$.

Answer: C

Factoring with leading coefficient $\neq 1$

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$$2x^2 - 3x - 2 = 0$$

Which of the following gives all values of x that satisfy the equation above?

(A) $x = -\dfrac{1}{2}$ only

(B) $x = 2$ only

(C) $x = -\dfrac{1}{2}$ and $x = 2$

(D) $x = \dfrac{1}{2}$ and $x = -2$

Solution:

Factor by finding two numbers whose product is $2\cdot(-2) = -4$ and whose sum is -3: those numbers are -4 and 1.

$$2x^2 - 4x + x - 2 = 2x(x-2) + 1(x-2) = (2x+1)(x-2) = 0$$

$$2x + 1 = 0 \;\Rightarrow\; x = -\tfrac{1}{2} \qquad \text{or} \qquad x - 2 = 0 \;\Rightarrow\; x = 2$$

Answer: (C)

Sum of Roots

- What is the sum of the solutions to the equation $2x^2 - 12x + 10 = 0$?

Solution:
For any quadratic equation $ax^2 + bx + c = 0$, the sum of the solutions is given by the formula $-\dfrac{b}{a}$.
Here, $a = 2$ and $b = -12$.

$$\text{Sum} = -\frac{-12}{2} = \frac{12}{2} = 6$$

Alternatively, you can factor the equation:

$$2(x^2 - 6x + 5) = 0 \Rightarrow 2(x - 5)(x - 1) = 0$$

The solutions are $x = 5$ and $x = 1$. The sum is $5 + 1 = 6$.
Answer: 6

- In the xy-plane, the graph of $y = x^2 - 9$ intersects the graph of $y = 5x + 5$ at two points. What is the sum of the x-coordinates of these two points?

Solution:
To find the intersection points, set the two equations equal to each other:

$$x^2 - 9 = 5x + 5$$

Move all terms to one side to form a quadratic equation:

$$x^2 - 5x - 14 = 0$$

To find the sum of the x-coordinates, use the sum of roots formula ($-\frac{b}{a}$):

$$\text{Sum} = -\frac{-5}{1} = 5$$

Alternatively: Factoring gives $(x - 7)(x + 2) = 0$, so $x = 7$ and $x = -2$, which also sum to 5.
Answer: 5

Factored vs. vertex form; identifying x-intercepts

- $$y = x^2 - 4x + 3$$

The graph of the equation above is a parabola in the xy-plane. Which of the following is an equivalent form of the equation that displays the x-intercepts of the parabola as constants or coefficients?

(A) $y = (x-2)^2 - 1$

(B) $y = (x-1)(x-3)$

(C) $y = (x+1)(x+3)$

(D) $y = x(x-4) + 3$

Solution:

Factor the right-hand side:

$$x^2 - 4x + 3 = (x-1)(x-3)$$

Setting $y = 0$ gives the x-intercepts directly from the factors:

$$(x-1)(x-3) = 0 \Rightarrow x = 1 \text{ and } x = 3$$

Why not (A)? Vertex form $(x-2)^2 - 1$ displays the vertex $(2, -1)$, not the x-intercepts.
Why not (D)? $x(x-4) + 3$ expands back to $x^2 - 4x + 3$ but the constants shown $(4, 3)$ do not directly reveal the intercepts.

Answer: (B)

Constants in Quadratic Equations

- In the equation $y = ax^2 + c$, the graph passes through $(0, 5)$ and $(2, 13)$. What is the value of a?

Solution

Using $(0, 5)$: $5 = a(0)^2 + c \Rightarrow c = 5$.
Using $(2, 13)$ and $c = 5$: $13 = a(2)^2 + 5 \Rightarrow 13 = 4a + 5 \Rightarrow 8 = 4a \Rightarrow a = 2$.
Answer: 2

Problems adapted from the College Board SAT Question Bank and released SAT practice tests.