Identifying the Vertex

• The function f is defined by $f(x) = x^2 - 4x + 3$. Which of the following is an equivalent form that displays the vertex of the parabola as constants or coefficients?

(A) $f(x) = (x-1)(x-3)$

(B) $f(x) = (x-2)^2 + 1$

(C) $f(x) = (x-2)^2 - 1$

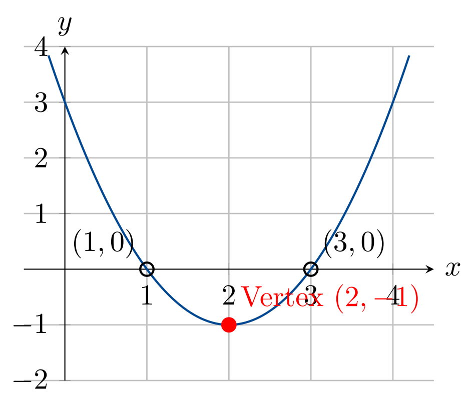
(D) $f(x) = (x+2)^2 - 1$

Solution

Complete the square:

$$f(x) = x^2 - 4x + 3 = (x^2 - 4x + 4) - 4 + 3 = (x-2)^2 - 1$$

The vertex is $(2, -1)$, visible directly from vertex form.



Factored Form and Intercepts

• A quadratic function $g(x)$ is defined by $g(x) = a(x+2)(x-4)$. If the graph of $y = g(x)$ passes through the point $(1, -18)$, what is the value of a?

Solution

Substitute $(1, -18)$ into $g(x) = a(x+2)(x-4)$:
$-18 = a(1+2)(1-4)$
$-18 = a(3)(-3) \Rightarrow -18 = -9a \Rightarrow a = 2$.

Answer: 2

• The graph of $y = ax^2 + bx + c$ passes through the points $(-1, 0)$, $(3, 0)$, and $(0, -3)$. What is the value of a?

(A) -1

(B) 1

(C) -3

(D) 3

Solution

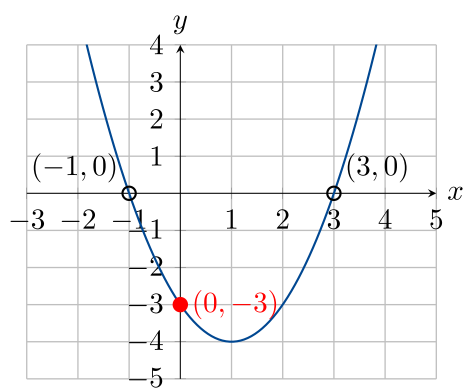
Since $x = -1$ and $x = 3$ are x-intercepts:

$$y = a(x+1)(x-3)$$

Use the point $(0, -3)$:

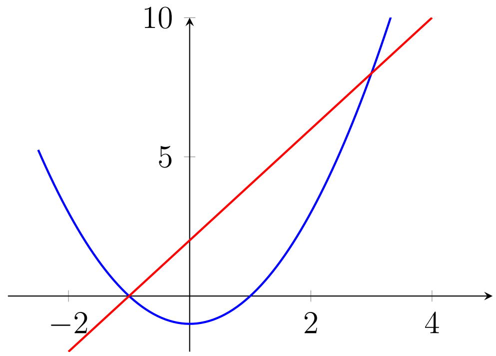
$$-3 = a(1)(-3) = -3a \;\Rightarrow\; a = 1$$

Answer is (B)



Intersection Points

• In the xy-plane, the line $y = 2x + 2$ intersects the parabola $y = x^2 - 1$ at two points. What is the x-coordinate of the point of intersection that lies in the first quadrant?



Solution

Set equations equal: $x^2 - 1 = 2x + 2$.
$x^2 - 2x - 3 = 0 \Rightarrow (x - 3)(x + 1) = 0$.
Solutions are $x = 3$ and $x = -1$. The first quadrant point has $x = 3$.

Answer: 3

• The graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = x + 1$ are shown in the xy-plane. At how many points do the two graphs intersect, and what are the x-coordinates of those points?

(A) They intersect at $x = -1$ and $x = 4$

(B) They intersect at $x = 1$ and $x = -4$

(C) They intersect at $x = -1$ only

(D) They do not intersect

Solution

Set $f(x) = g(x)$:

$$x^2 - 2x - 3 = x + 1$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0 \;\Rightarrow\; x = 4 \text{ or } x = -1$$

The two graphs intersect at $(-1, 0)$ and $(4, 5)$.
Answer: (A)

Projectile Motion

• A ball is thrown upward. Its height h, in meters, t seconds after it is thrown is modeled by $h(t) = -5t^2 + 20t + 2$. What is the maximum height, in meters, reached by the ball?

Solution

The maximum occurs at the vertex $t = -b/2a = -20/(2 \times -5) = 2$ seconds.
$h(2) = -5(2)^2 + 20(2) + 2 = -20 + 40 + 2 = 22$.

Answer: 22

• The function $f(x) = -2(x-3)^2 + 8$ represents the height (in feet) of a ball x seconds after being thrown. What is the maximum height, and when does it occur?

(A) Maximum height of 8 ft at $x = 3$ s

(B) Maximum height of 3 ft at $x = 8$ s

(C) Maximum height of 8 ft at $x = -3$ s

(D) Maximum height of -8 ft at $x = 3$ s

Solution

The function is in vertex form $f(x) = a(x-h)^2 + k$ with $a = -2$, $h = 3$, $k = 8$.
Since $a < 0$ the parabola opens downward, so the vertex is a **maximum**.

$$\text{Maximum value} = k = 8 \text{ ft, occurring at } x = h = 3 \text{ s.}$$

Vertex Form

• The graph of $y = x^2$ is translated 4 units to the right and 3 units up. Which of the following equations represents the resulting parabola?

A) $y = (x + 4)^2 + 3$
B) $y = (x - 4)^2 + 3$
C) $y = (x - 4)^2 - 3$
D) $y = (x + 4)^2 - 3$

Solution

Vertex form is $y = a(x - h)^2 + k$. Right 4 units means $h = 4$; up 3 units means $k = 3$. Thus, $y = (x - 4)^2 + 3$.

Answer: B

• If $f(x) = 2x^2 - 8x + 6$, what is the x-coordinate of the vertex?

(A) -2

(B) 2

(C) 4

(D) -4

Solution

Use the vertex formula $x = -\dfrac{b}{2a}$:

$$x = -\frac{-8}{2(2)} = \frac{8}{4} = 2$$

Alternatively, factor out and complete the square:

$$f(x) = 2(x^2 - 4x) + 6 = 2(x^2 - 4x + 4 - 4) + 6 = 2(x - 2)^2 - 2$$

Vertex is at $(2, -2)$.

Answer: (B)

• The function $f(x) = x^2 + bx + c$ has x-intercepts at $x = -2$ and $x = 8$. What is the x-coordinate of the vertex of the graph of f?
Solution

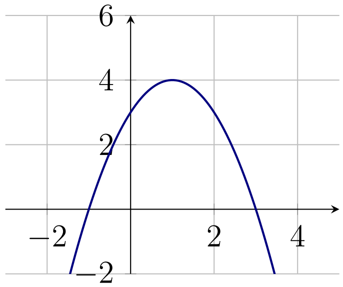
The vertex x-coordinate is the midpoint of the roots:
$h = \frac{-2+8}{2} = \frac{6}{2} = 3.$

Answer: 3

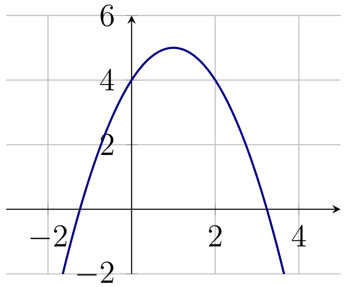
Graphing from an Equation

- Which of the following is the graph of $y = -(x-1)^2 + 4$?

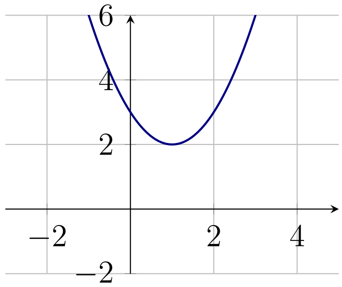
A)



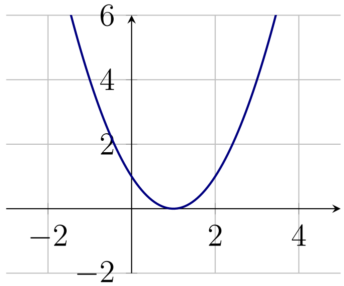
B)



C)



D)

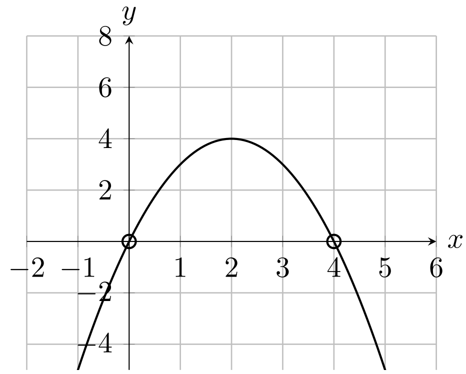


Solution

The vertex should be at $(1, 4)$ and the parabola must open downward because $a = -1$.

Answer: A

- The graph of the quadratic function g is shown below. Which equation could represent g?



(A) $g(x) = (x-2)^2 + 4$

(B) $g(x) = -(x-2)^2 + 4$

(C) $g(x) = (x+2)^2 - 4$

(D) $g(x) = -(x+2)^2 + 4$

Solution

From the graph the parabola opens downward (eliminates A and C) and has vertex at $(2, 4)$.

$$g(x) = -(x-2)^2 + 4$$

Verify zeros: $-(x-2)^2 + 4 = 0 \Rightarrow (x-2)^2 = 4 \Rightarrow x = 0$ or $x = 4$.

Answer: (B)

Optimization in a real-world profit model

The profit P (in thousands of dollars) of a company is modeled by

$$P(x) = -x^2 + 10x - 16$$

where x is the number of units sold (in hundreds). What is the maximum profit, and for how many units does it occur?

(A) Maximum profit of \$9,000 at $x = 5$ hundred units

(B) Maximum profit of \$16,000 at $x = 10$ hundred units

(C) Maximum profit of \$25,000 at $x = 5$ hundred units

(D) Maximum profit of \$9,000 at $x = 10$ hundred units

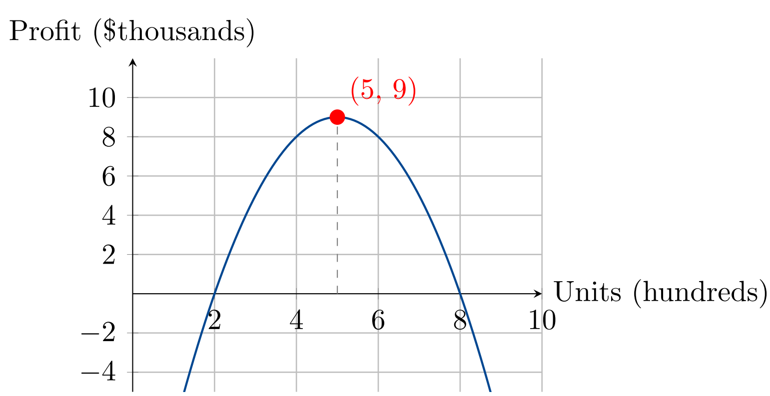
Solution

Find the vertex using $x = -\dfrac{b}{2a}$:

$$x = -\frac{10}{2(-1)} = 5$$

$$P(5) = -(5)^2 + 10(5) - 16 = -25 + 50 - 16 = 9$$

Maximum profit is \$9,000 (since P is in thousands) at $x = 5$ hundred units.



Answer: (A)

Problems adapted from the College Board SAT Question Bank and released SAT practice tests.