Equivalent Expressions (Basic)

- Which of the following is equivalent to $x^{\frac{2}{3}}$ for all $x > 0$?

  A) $\sqrt{x^3}$

  B) $\sqrt[3]{x^2}$

  C) $\sqrt[3]{x}$

  D) $x^2 + x^3$

Solution:
By the definition of rational exponents, $x^{\frac{a}{b}} = \sqrt[b]{x^a}$. Therefore, $x^{\frac{2}{3}} = \sqrt[3]{x^2}$.
The correct answer is B.

Simple Radical Equation

- If $\sqrt{2x+6} = 4$, what is the value of x?

  A) 1

  B) 5

  C) 10

  D) 16

Solution:

$$\begin{aligned} \sqrt{2x+6} &= 4 \\ (\sqrt{2x+6})^2 &= 4^2 \\ 2x+6 &= 16 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

Check: $\sqrt{2(5)+6} = \sqrt{16} = 4$.
The correct answer is B.

Extraneous Solutions

- What is the solution set for the equation $\sqrt{x+10} = x - 2$?

A) $\{-1, 6\}$

B) $\{6\}$

C) $\{-1\}$

D) $\{2, 10\}$

Solution:
Square both sides:

$$\begin{aligned} x + 10 &= (x-2)^2 \\ x + 10 &= x^2 - 4x + 4 \\ 0 &= x^2 - 5x - 6 \\ 0 &= (x-6)(x+1) \end{aligned}$$

Potential solutions are $x = 6$ and $x = -1$.
Check $x = 6$: $\sqrt{6+10} = 6 - 2 \Rightarrow \sqrt{16} = 4$ (True).
Check $x = -1$: $\sqrt{-1+10} = -1 - 2 \Rightarrow \sqrt{9} = -3$ (False).
Only $x = 6$ is a solution.
The correct answer is **B**.

- Solve for x: $\sqrt{x+5} = x - 1$.

Solution:

$$\begin{aligned} x + 5 = (x-1)^2 &= x^2 - 2x + 1 \\ 0 = x^2 - 3x - 4 &= (x-4)(x+1) \end{aligned}$$

$x = 4$ or $x = -1$.

Check $x = -1$:
LHS $= 2$, RHS $= -2$. Extraneous! Only solution acceptable is: $x = 4$

Combining Radicals with different indices

- For $x > 0$, which of the following is equivalent to $\dfrac{\sqrt{x^3}}{\sqrt[4]{x}}$?

A) $x^{5/4}$

B) $x^{3/4}$

C) $x^{1/2}$

D) x^2

Solution:
Convert radicals to rational exponents:

$$\frac{x^{3/2}}{x^{1/4}} = x^{(3/2-1/4)} = x^{(6/4-1/4)} = x^{5/4}$$

The correct answer is A.

Isolating a Radical

- If $5 + \sqrt{n-3} = 12$, what is the value of n?

  A) 10

  B) 46

  C) 52

  D) 64

Solution:
Isolate the radical:

$$\begin{aligned} \sqrt{n-3} &= 7 \\ (\sqrt{n-3})^2 &= 7^2 \\ n-3 &= 49 \\ n &= 52 \end{aligned}$$

The correct answer is C.

- Solve $\sqrt{4x-1} + 3 = 2x$.

Solution:

$$\begin{aligned} \sqrt{4x-1} &= 2x-3 \quad (x \geq 1.5) \\ 4x-1 &= 4x^2 - 12x + 9 \\ 2x^2 - 8x + 5 &= 0 \\ x &= \frac{8 \pm \sqrt{24}}{4} = \frac{4 \pm \sqrt{6}}{2} \end{aligned}$$

Only $x = \dfrac{4+\sqrt{6}}{2}$ satisfies the original equation.

Simplifying Higher Roots

- Which of the following is equivalent to $\sqrt[3]{27x^6y^9}$?

  A) $3x^2y^3$

  B) $9x^2y^3$

  C) $3x^3y^6$

  D) $27x^2y^3$

Solution:
Distribute the cube root:

$$\sqrt[3]{27} \cdot \sqrt[3]{x^6} \cdot \sqrt[3]{y^9} = 3 \cdot x^{6/3} \cdot y^{9/3} = 3x^2y^3$$

The correct answer is A.

Function Context

- In the function $f(x) = \sqrt{k - x}$, if $f(-5) = 4$, what is the value of k?

  A) 1

  B) 9

  C) 11

  D) 21

Solution:
Substitute $x = -5$ and $f(x) = 4$:

$$\begin{aligned} 4 &= \sqrt{k - (-5)} \\ 4 &= \sqrt{k + 5} \\ 16 &= k + 5 \\ k &= 11 \end{aligned}$$

The correct answer is C.

- Which of the following is the domain of $f(x) = \sqrt{4 - 2x}$.

  A) $(2, \infty)$

  B) $[2, \infty))$

  C) $(-\infty, 2]$

  D) $(-\infty, 2)$

Solution:

$$\begin{aligned} 4 - 2x &\geq 0 \\ x &\leq 2 \end{aligned}$$

Domain: $(-\infty, 2]$
The correct answer is C.

Nested Radicals / Variables in Exponents

- If $x^a = \sqrt{\sqrt[3]{x}}$, what is the value of a?

  A) 1/6

  B) 1/5

  C) 1/3

  D) 5/6

Solution:
Convert the radicals step-by-step:

$$\sqrt{\sqrt[3]{x}} = \sqrt{x^{1/3}} = (x^{1/3})^{1/2} = x^{(1/3 \cdot 1/2)} = x^{1/6}$$

So $a = 1/6$. The correct answer is A.

Radical Geometry/Context

- A square has an area of 50 square units. What is the length of its diagonal?

  A) $5\sqrt{2}$

  B) 10

  C) $10\sqrt{2}$

  D) 25

Solution:
If Area $s^2 = 50$, then side length $s = \sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$.
The diagonal d of a square is $s\sqrt{2}$:

$$d = (5\sqrt{2})\sqrt{2} = 5 \cdot 2 = 10$$

The correct answer is B.

Relationship between variables

- If $y = a\sqrt{x}$ and $y = 12$ when $x = 9$, what is the value of y when $x = 4$?

  A) 4

  B) 8

  C) 16

  D) 18

Solution:
First, find a:

$$12 = a\sqrt{9} \Rightarrow 12 = 3a \Rightarrow a = 4$$

Now find y for $x = 4$:

$$y = 4\sqrt{4} = 4(2) = 8$$

The correct answer is B.