Infinitely many solutions

• In the system of equations below

$$\begin{cases} 5x - 2y = 3 \\ ax + by = 6 \end{cases}$$

a and b are constants. If the system has infinitely many solutions, what is the value of $a + b$?

Solution:
Transform each equation in the slope intercept form:

$$\begin{cases} y = \frac{5}{2}x - \frac{3}{2} \\ y = -\frac{a}{b}x + \frac{6}{b} \end{cases}$$

Since this system of equation has infinitely many solutions, this means that the two lines which represents the equations must have the same slope and same y-intercept, therefore

$$\frac{5}{2} = -\frac{a}{b} \text{ and } -\frac{3}{2} = \frac{6}{b}$$

Solving second proportion for b we get $-3b = 12$ or $b = -4$. Solving for a we get $5b = -2a$ or $-20 = -2a$ so $a = 10$ and $a + b = 6$

• The system of equations below has infinitely many solutions. What is the value of $a + b$?

$$ax + 3y = 12$$
$$2x + y = 4$$

Solution:
For a system to have infinitely many solutions, the two equations must be equivalent (one is a multiple of the other). Observe the constants: the first equation has 12 and the second has 4. $12 \div 4 = 3$. Multiply the entire second equation by 3:

$$3(2x + y = 4) \implies 6x + 3y = 12$$

Comparing $6x + 3y = 12$ to $ax + 3y = 12$, we see $a = 6$. In this case, the equations are identical, so b is not explicitly in the problem, but typically b represents the constant. If the second equation was $2x + y = b$, then $12 = 3b$, so $b = 4$. Sum: $6 + 4 = 10$.
Answer: 10

- In the system below, for what value of a will the system have infinitely many solutions?

$$\frac{1}{2}x + \frac{1}{3}y = 5$$
$$ax + 2y = 30$$

Solution:
To make the y coefficients match, look at the first equation: $\frac{1}{3}y$ needs to become $2y$. Multiply the first equation by 6:

$$6\left(\frac{1}{2}x + \frac{1}{3}y = 5\right) \implies 3x + 2y = 30$$

Comparing this to $ax + 2y = 30$, we see that a must be 3.
Answer: 3

One solution

- $$\begin{cases} ax + 2y = 5 \\ 3x - 6y = 20 \end{cases}$$

In the system of equations above, a is a constant. If the system has one solution, which of the following can NOT be a value of a?

(A) -1
(B) 3/4
(C) 1
(D) 3

Solution:

Transform each equation in slope intercept form $\begin{cases} y = -\frac{a}{2}x + \frac{5}{2} \\ y = \frac{1}{2}x - \frac{10}{3} \end{cases}$

Since this system of equation has one solution then slopes of two lines must be different that is $-\frac{a}{2} \neq \frac{1}{2}$ therefore $a \neq -1$

No Solution (Parallel Lines)

• In the system of equations below, k is a constant. If the system has no solution, what is the value of k?

$$4x - 5y = 7$$
$$kx - 10y = 8$$

Solution:
A system of linear equations has no solution if the lines are parallel and have different y-intercepts. Parallel lines must have proportional coefficients for x and y:

$$\frac{4}{k} = \frac{-5}{-10}$$

Simplify the right side:

$$\frac{4}{k} = \frac{1}{2}$$

Cross-multiply to solve for k:

$$k = 8$$

Answer: 8

• For which values of k, will the system of equations

$$\begin{cases} 2x - 5y = 8 \\ ky + 4x = 17 \end{cases}$$

has no solution?

Solution:
Transform each equation in slope intercept form:

$$\begin{cases} y = \frac{2}{5}x - \frac{8}{5} \\ y = -\frac{4}{k}x + \frac{17}{k} \end{cases}$$

Since the system of equation has no solution, the two lines which represents the equations are parallel therefore must have the same slope, thus $\frac{2}{5} = -\frac{4}{k}$.

Solving for k by cross multiplication we have $-2k = 20$ or $k = -10$

Contextual Word Problem

• A local theater sells adult tickets for \$15 and child tickets for \$10. On Saturday, the theater sold 150 tickets total and collected \$1,900. How many adult tickets (a) were sold?

Solution:
Create a system of equations:

1. $a + c = 150$ (Total tickets)
2. $15a + 10c = 1900$ (Total revenue)

Solve for c in the first: $c = 150 - a$. Substitute into the second:

$$15a + 10(150 - a) = 1900$$

$$15a + 1500 - 10a = 1900$$

$$5a = 400$$

$$a = 80$$

Answer: 80

• A school sold adult tickets for \$8 and student tickets for \$5. A total of 200 tickets were sold for \$1,390. How many adult tickets were sold?

(A) 63

(B) 97

(C) 130

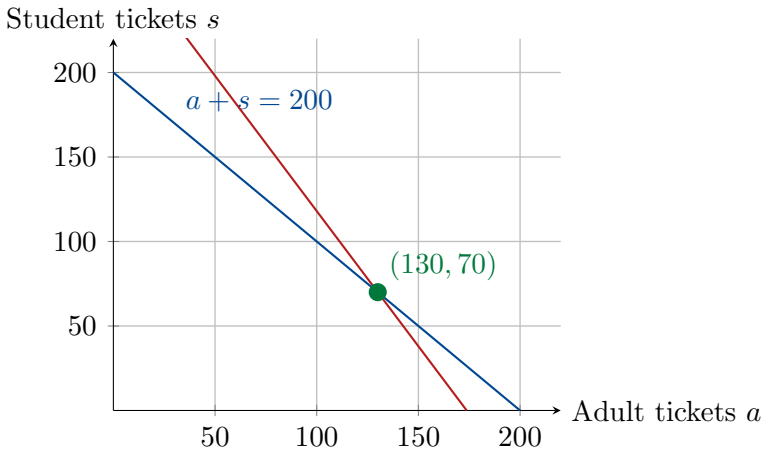
(D) 150

Solution:
Let a = adult tickets, s = student tickets.

$$\begin{cases} a + s = 200 \\ 8a + 5s = 1{,}390 \end{cases}$$

From the first equation: $s = 200 - a$. Substitute:

$$8a + 5(200 - a) = 1{,}390 \;\Rightarrow\; 8a + 1{,}000 - 5a = 1{,}390 \;\Rightarrow\; 3a = 390 \;\Rightarrow\; a = 130$$


Student tickets s
200
150
100
50
$a + s = 200$
(130, 70)
50
100
150
200
Adult tickets a

Answer: (C)

Solving for an Expression

• If $3x + 7y = 22$ and $2x + 6y = 12$, what is the value of $12x + 12y$?

(A) 34
(B) 58
(C) 120
(D) 156

Solution:

Instead of solving entire system of equations we can subtract the two equations which yields.
$3x + 7y = 22$
$-(2x + 6y = 12)$

$x + y = 10$

Multiply both sides of last equation by 12 we get $12(x + y) = 12(10)$ or $12x + 12y = 120$

• If (x, y) is the solution to the system below, what is the value of $x - y$?

$$2x - 3y = 14$$
$$x + 2y = -7$$

Solution:
Multiply the second equation by 2: $2x + 4y = -14$. Subtract the first equation from this new equation:

$$(2x + 4y) - (2x - 3y) = -14 - 14$$
$$7y = -28 \implies y = -4$$

Plug $y = -4$ into $x + 2y = -7$:

$$x + 2(-4) = -7 \implies x - 8 = -7 \implies x = 1$$

Calculate $x - y$: $1 - (-4) = 5$.
Answer: 5

Problems adapted from the College Board SAT Question Bank and released SAT practice tests.