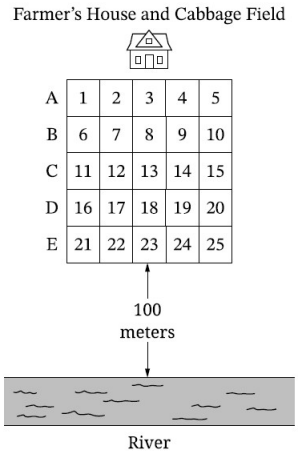
• Aphids are tiny insects that feed on plants such as cabbage plants. A farmer wants to reduce the number of aphids in a cabbage field. A river is located 100 meters south of the cabbage field. The farmer divides the field into 25 regions of equal size, as shown in the diagram. Each region has approximately the same number of cabbage plants.



The farmer would like to estimate the proportion of cabbage plants in the field that are affected by aphids and believes that the extent of aphid damage is greater for the regions in the cabbage field closer to the river. To obtain the estimate, the farmer is considering three sampling methods.

- **Sampling method I:** Select region 3, which is closest to the farmer's house and farthest from the river. Examine every cabbage plant in the region for aphid damage.
- **Sampling method II:** Randomly select one row (A, B, C, D, or E). For every region in the selected row, examine every cabbage plant for aphid damage.
- **Sampling method III:** Randomly select one region from each of rows A, B, C, D, and E. For each selected region, examine every cabbage plant for aphid damage.

(A). Explain whether sampling method I is an appropriate sampling method for the farmer to use to estimate the proportion of cabbage plants in the field that are damaged by aphids.

(B). Using sampling method II, the farmer randomly selected row E and examined every cabbage plant in row E. If the farmer's belief is correct, determine whether the selection of row E is likely to provide an overestimate or an underestimate of the proportion of cabbage plants in the field that are damaged by aphids. Justify your answer.

(C). Using the information provided in the diagram of the cabbage field, describe how to implement sampling method III, which requires a random selection of one region from each of rows A, B, C, D, and E.

Solution:

(A) Sampling method I is not an appropriate sampling method for the farmer to use to estimate the proportion of cabbage plants in the field that are affected by aphids. Sampling method I is a convenience sample where region 3 is not selected randomly. If the farmer's belief is correct, there may be fewer cabbage plants that are affected by aphids in region 3 than in most other regions of the cabbage field because region 3 is in the row farthest from the river. This may lead to an underestimate of the proportion of cabbage plants in the field that are damaged by aphids

(B) The selection of row E is likely to provide an overestimate of the proportion of all cabbage plants in the field that are damaged by aphids. If the farmer's belief that the extent of aphid damage is greater for the regions in the cabbage field closer to the river is correct, then row E, which is the row of regions located closest to the river, is likely to have a greater proportion of cabbage plants damaged by aphids than regions farther from the river.

(C) The farmer should write the region numbers from row A, 1 through 5, onto same-size slips of paper, then put the numbers into a hat, mix well, and select one of the numbers. The farmer should repeat this process for the region numbers of each of the other rows (i.e., row B, 6 through 10; row C, 11 through 15; row D, 16 through 20; row E, 21 through 25) and select one number from each row. This process will result in the selection of one region from each row. The farmer will examine every cabbage plant in each of the selected regions for aphid damage to determine the proportion of cabbage plants in the selected regions that are damaged by aphids.

Alternative Solution: The farmer should use a random number generator to generate one two-digit integer from 01 to 05, one two-digit integer from 06 to 10, one two-digit integer from 11 to 15, one two-digit integer from 16 to 20, and one two-digit integer from 21 to 25. For each integer selected, the farmer should select the corresponding numbered region and examine every cabbage plant in each of the selected regions for aphid damage to determine the proportion of cabbage plants in the selected regions that are damaged by aphids.

• A car maker produces four different models of cars: A, B, C, and D. A group of researchers is investigating which model of car has the longest distance traveled per gallon of gas (mileage). Higher mileage is considered better than lower mileage. The researchers will conduct a study in which they contact several owners of each model of car and ask them to estimate their mileage.

(A) Is this an observational study or an experiment? Justify your answer in context.

Model D has an autopilot feature, in which the car controls its own motion with human supervision. James owns a Model D car and will investigate whether using the autopilot feature results in higher mileage than not using the autopilot. James will drive his car on 70 different days to and from work, using the same route at the same time each day. James will record the mileage each day.

(B) James will use a completely randomized design to conduct his investigation. Describe an appropriate method James could use to randomly assign the two treatments, driving using the autopilot feature and driving with out using the autopilot feature, to 35 days each.

(C) After the investigation was completed, James verified that the conditions for inference were met and conducted a hypothesis test. He discovered the mean mileage when using the autopilot feature was significantly higher than the mean mileage when not using the autopilot feature.

James is a member of a Model D club with thousands of members who all drive Model D cars. He will give a presentation at a Model D club members' meeting later this year and would like to state that the results of his hypothesis test apply to all Model D cars in his club. Another member of the club who is a statistician tells James his findings do not apply to all Model D cars in the club. What change would James need to make to his original study to be able to generalize to all Model D cars in the club?

Solution:
(A) This is an observational study. The researchers had the car owners estimate their mileage. The car owners were not randomly assigned a car model, so no treatment was imposed.

(B) Number the days in the experiment from 1 to 70. Using a random number generator, generate 35 unique integers from 1 to 70, inclusive. Assign the days with those 35 unique integers for James to drive the car with autopilot and assign the remaining 35 days for James to drive the car without autopilot.

(Alternative solution) Using 70 equally sized slips of paper, label 35 "with autopilot" and 35 "without autopilot." Mix the slips of paper in a bag. Each day for the 70 days, select a slip of paper (without replacement) to determine the driving method for that day.

(C) In order to generalize his findings to all Model D cars in his club, he would need to randomly select Model D cars from the club. He would then need to carry out a similar study using the Model D cars that were randomly sampled from the club.

• A developer wants to know whether adding fibers to concrete used in paving driveways will reduce the severity of cracking, because any driveway with severe cracks will have to be repaired by the developer. The developer conducts a completely randomized experiment with 60 new homes that need driveways. Thirty of the driveways will be randomly assigned to receive concrete that contains fibers, and the other 30 driveways will receive concrete that does not contain fibers. After one year, the developer will record the severity of cracks in each driveway on a scale of 0 to 10, with 0 representing not cracked at all and 10 representing severely cracked.

(A) Based on the information provided about the developer's experiment, identify each of the following.

- Experimental units
- Treatments
- Response variable

(B) Describe an appropriate method the developer could use to randomly assign concrete that contains fibers and concrete that does not contain fibers to the 60 driveways.

Suppose the developer finds that there is a statistically significant reduction in the mean severity of cracks in driveways using the concrete that contains fibers compared to the driveways using concrete that does not contain fibers.

(C) In terms of the developer's conclusion, what is the benefit of randomly assigning the driveways to either the concrete that contains fibers or the concrete that does not contain fibers?

Solution:

(A) Experimental units: 60 driveways.

Treatments: Concrete with fibers and concrete without fibers.

Response variable: Rating of the severity of the cracks after one year, on a scale of 0 to 10.

(B) Number the 60 driveways from 01 to 60. Using a random number generator, generate two-digit integers between 01 and 60. Ignore 00 and any number greater than 60 until 30 unique numbers are obtained. Assign the driveways with those 30 unique numbers to receive concrete with fibers and the remaining 30 driveways to receive concrete without fibers.

(C) The results were statistically significant, and because the driveways were randomly assigned to either the concrete with the fibers or the concrete without the fibers, there is evidence the treatment (type of concrete) caused the response (rating of severity of cracks).

• A dermatologist will conduct an experiment to investigate the effectiveness of a new drug to treat acne. The dermatologist has recruited 36 pairs of identical twins. Each person in the experiment has acne and each person in the experiment will receive either the new drug or a placebo. After each person in the experiment uses either the new drug or the placebo for 2 weeks, the dermatologist will evaluate the improvement in acne severity for each person on a scale from 0 (no improvement) to 100 (complete cure).

(A) Identify the treatments, experimental units, and response variable of the experiment.

- Treatments:
- Experimental units:
- Response variable:

Each twin in the experiment has a severity of acne similar to that of the other twin. However, the severity of acne differs from one twin pair to another.

(B) For the dermatologist's experiment, describe a statistical advantage of using a matched-pairs design where twins are paired rather than using a completely randomized design.

(C) For the dermatologist's experiment, describe how the treatments can be randomly assigned to people using a matched-pairs design in which twins are paired.

Solutions:
(A) Treatments: New drug, placebo.
Experimental units: The 72 people who receive the new drug or placebo.
Response variable: Improvement in acne severity

(B) Improvement scores will vary due to many factors, including initial acne severity, what treatment is received, and other variables such as diet and genetics. Because the pairs of twins are similar in initial acne severity, pairing allows for the variation in improvement scores due to the treatment received to be distinguished from variation due to initial acne severity, unlike in a completely randomized design. Consequently, using the matched-pairs design will provide a more precise estimate of the mean difference in improvement in acne severity for the new drug compared to the placebo and make it easier to find convincing evidence that the new drug is better, if it really is better.

(C) For each pair of twins, label one person as twin A and label the other person as twin B. For each pair of twins, toss a coin. If the coin lands on heads, twin A gets the placebo and twin B gets the active drug. If the coin lands on tails, twin A gets the active drug and twin B gets the placebo.
OR
Label the members of each pair of twins as "Twin 1" and "Twin 2." Using a random number generator, generate an integer from 1 to 2. Give the drug to the twin whose number is selected and the placebo to the twin whose number is not selected. Repeat for all pairs of twins.
OR

Label 1 notecard "A" and another notecard "B." For each pair of twins, shuffle the cards and give one card to each twin. The twin who gets "A" receives the drug and the twin who gets "B" receives the placebo.

Problems adapted from the College Board released tests.