

• A jewelry company uses a machine to apply a coating of gold on a certain style of necklace. The amount of gold applied to a necklace is approximately normally distributed. When the machine is working properly, the amount of gold applied to a necklace has a mean of 300 milligrams (mg) and standard deviation of 5 mg.

- (A) A necklace is randomly selected from the necklaces produced by the machine. Assuming that the machine is working properly, calculate the probability that the amount of gold applied to the necklace is between 296 mg and 304 mg.

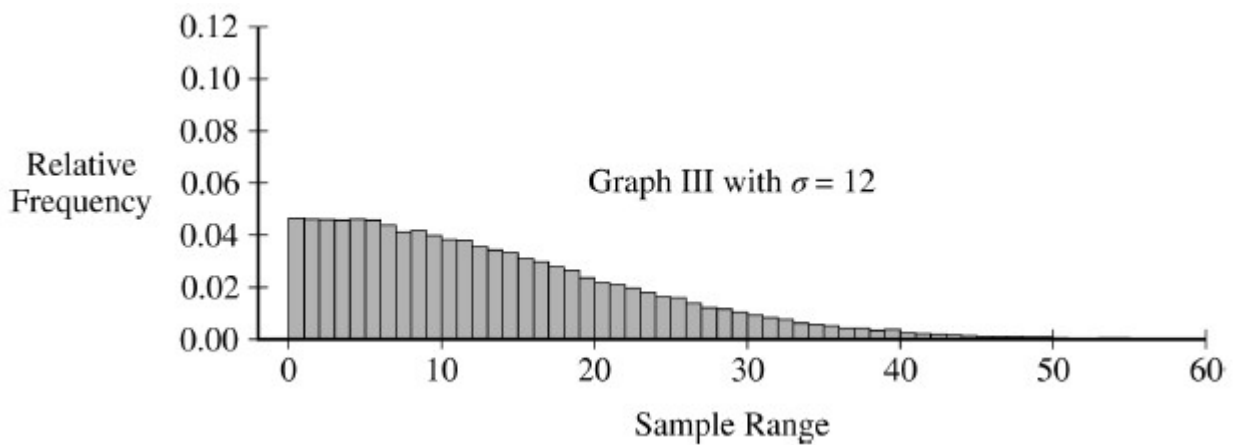
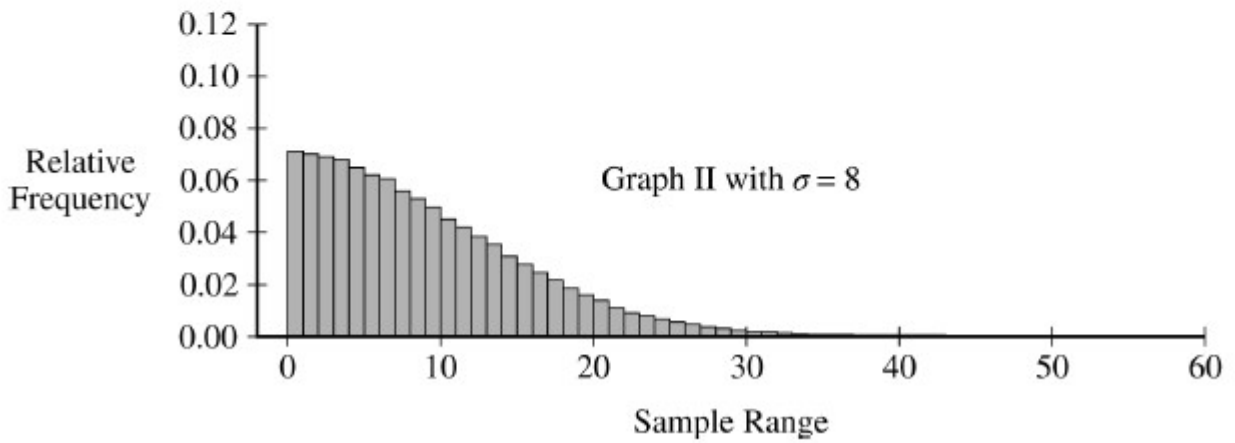
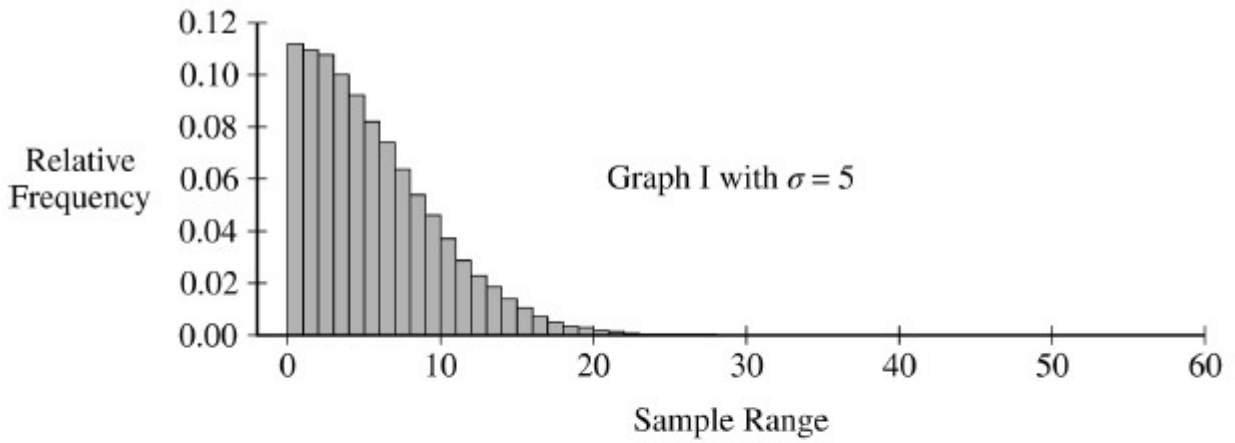
The jewelry company wants to make sure the machine is working properly. Each day, Cleo, a statistician at the jewelry company, will take a random sample of the necklaces produced that day. Each selected necklace will be melted down and the amount of the gold applied to that necklace will be determined. Because a necklace must be destroyed to determine the amount of gold that was applied, Cleo will use random samples of size $n = 2$ necklaces.

Cleo starts by considering the mean amount of gold being applied to the necklaces. After Cleo takes a random sample of $n = 2$ necklaces, she computes the sample mean amount of gold applied to the two necklaces.

- (B) Suppose the machine is working properly with a population mean amount of gold being applied of 300 mg and a population standard deviation of 5 mg.
- Calculate the probability that the sample mean amount of gold applied to a random sample of $n = 2$ necklaces will be greater than 303 mg.
 - Suppose Cleo took a random sample of $n = 2$ necklaces that resulted in a sample mean amount of gold applied of 303 mg. Would that result indicate that the population mean amount of gold being applied by the machine is different from 300 mg? Justify your answer without performing an inference procedure.

Now, Cleo will consider the variation in the amount of gold the machine applies to the necklaces. Because of the small sample size, $n = 2$, Cleo will use the sample range of the data for the two randomly selected necklaces, rather than the sample standard deviation.

Cleo will investigate the behavior of the range for samples of size $n = 2$. She will simulate the sampling distribution of the range of the amount of gold applied to two randomly sampled necklaces. Cleo generates 100,000 random samples of size $n = 2$ independent values from a normal distribution with mean $\mu = 300$ and standard deviation $\sigma = 5$. The range is calculated for the two observations in each sample. The simulated sampling distribution of the range is shown in Graph I. This process is repeated using $\sigma = 8$, as shown in Graph II, and again using $\sigma = 12$, as shown in Graph III.



(C) Use the information in the graphs to complete the following.

- (i) Describe the sampling distribution of the sample range for random samples of size $n = 2$ from a normal distribution with standard deviation $\sigma = 5$, as shown in Graph I.

- (ii) Describe how the sampling distribution of the sample range for samples of size $n = 2$ changes as the value of the population standard deviation σ increases.

Recall that Cleo needs to consider both the mean and standard deviation of the amount of gold applied to necklaces to determine whether the machine is working properly. Suppose that one month later, Cleo is again checking the machine to make sure it is working properly. Cleo takes a random sample of 2 necklaces and calculates the sample mean amount of gold applied as 303 mg and the sample range as 10 mg.

- (D) Recall that the machine is working properly if the amount of gold applied to the necklaces has a mean of 300 mg and standard deviation of 5 mg.
- (i) Consider Cleo's range of 10 mg from the sample of size $n = 2$. If the machine is working properly with a standard deviation of 5 mg, is a sample range of 10 mg unusual? Justify your answer.
- (ii) Do Cleo's sample mean of 303 mg and range of 10 mg indicate that the machine is not working properly? Explain your answer.

Solutions:

(A) Let X represent the amount of gold applied to a necklace randomly selected from necklaces produced with this machine. The random variable X has an approximately normal distribution with mean 300 mg and standard deviation 5 mg.

Then,

$$\begin{aligned} P(296 < X < 304) &= P(X < 304) - P(X \leq 296) \\ &= P\left(Z < \frac{304 - 300}{5}\right) - P\left(Z \leq \frac{296 - 300}{5}\right) \\ &= P(Z < 0.8) - P(Z \leq -0.8) \\ &\approx 0.7881 - 0.2119 \approx 0.5763. \end{aligned}$$

OR normalcdf (lower = 296, upper = 304, mean = 300, standard deviation = 5).

(B)

(i) If the machine is working properly, and the sample mean amount of gold, \bar{X} , has a sampling distribution that follows a normal distribution with mean 300 mg and standard deviation

$$\frac{5}{\sqrt{2}} \approx 3.5355 \text{ mg, then}$$

$$\begin{aligned} P(\bar{X} > 303) &= P\left(Z > \frac{303 - 300}{\frac{5}{\sqrt{2}}}\right) \\ &\approx P(Z > 0.8485) \\ &\approx 0.198. \end{aligned}$$

OR normalcdf (lower 303, upper = ∞ , mean 300, standard deviation = $\frac{5}{\sqrt{2}}$)

(ii) Observing a sample mean amount of 303 mg would not provide convincing evidence that the population mean amount of gold being applied by the machine is something other than 300 mg because the probability of observing a sample mean that differs from 300 mg by 3 mg or more is large, around $0.198(2) = 0.396$.

(C)

(i) The sampling distribution of the sample range for random samples of size $n = 2$ from a normal distribution with standard deviation $\sigma = 5$ is skewed to the right. Almost all values of the simulated ranges are between 0 mg and about 25 mg and the center of the distribution is about 6 mg.

(ii) As the value of the population standard deviation increases, the variation (spread) in the distribution of the sample range increases and the mean of the distribution of the sample range also increases.

(D)

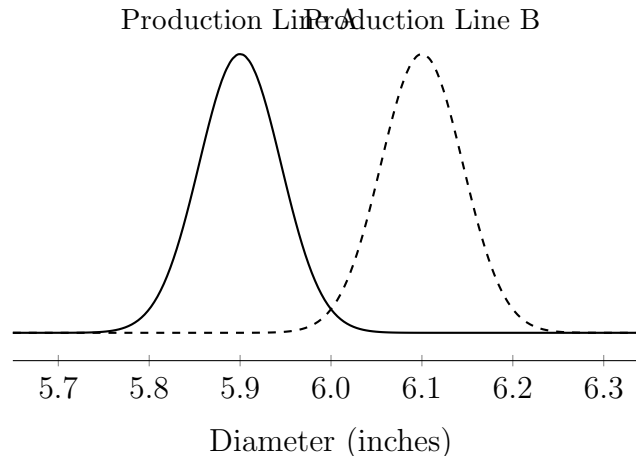
(i) No, a sample range of 10 mg is not unusual if the machine is working properly with a standard deviation of 5 mg.

Although observing a sample range around 10 mg or greater is much more likely if the population standard deviation is 8 mg or 12 mg than when the population standard deviation is 5 mg, the graph of the sampling distribution of the sample range for samples of size 2 from a normal distribution with $\sigma = 5$ mg indicates that 10 mg is not an unusual value for the range when $\sigma = 5$ mg. There is about a 20% chance that a random sample of two necklaces would yield a range of 10 mg or more when the machine is working properly.

(ii)

No, Cleo's sample mean of 303 mg and range of 10 mg do not indicate that the machine is not working properly. As noted in part (b-i), the probability that the sample mean would be equal to or greater than 303 mg when the machine is working properly is almost 20% so having a sample mean of 303 mg is not unusual. Furthermore, it is less than one standard deviation, $\frac{5}{\sqrt{2}} = 3.5355$ mg, away from 300 mg. As indicated in part (d-i), the probability of a range of 10 mg or greater when the population standard deviation is 5 mg is also about 20%, so not unusual. There is not statistically significant evidence to show the machine is not working properly.

- Corn tortillas are made at a large facility that produces 100,000 tortillas per day on each of its two production lines. The distribution of the diameters of the tortillas produced on production line A is approximately normal with mean 5.9 inches, and the distribution of the diameters of the tortillas produced on production line B is approximately normal with mean 6.1 inches. The figure below shows the distributions of diameters for the two production lines.



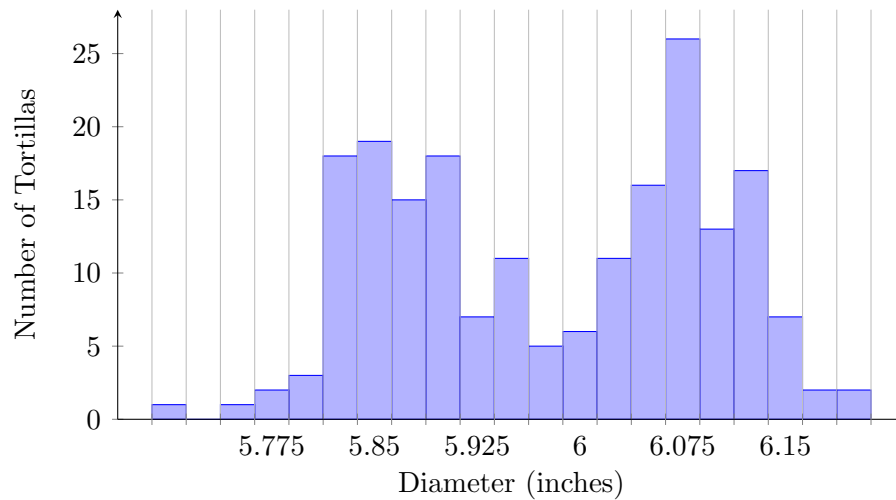
The tortillas produced at the factory are advertised as having a diameter of 6 inches. For the purpose of quality control, a sample of 200 tortillas is selected and the diameters are measured. From the sample of 200 tortillas, the manager of the facility wants to estimate the mean diameter, in inches, of the 200,000 tortillas produced on a given day. Two sampling methods have been proposed.

Method 1: Take a random sample of 200 tortillas from the 200,000 tortillas produced on a given day. Measure the diameter of each selected tortilla.

Method 2: Randomly select one of the two production lines on a given day. Take a random sample of 200 tortillas from the 100,000 tortillas produced by the selected production line. Measure the diameter of each selected tortilla.

(A) Will a sample obtained using Method 2 be representative of the population of all tortillas made that day, with respect to the diameters of the tortillas? Explain why or why not.

(B) The figure below is a histogram of 200 diameters obtained by using one of the two sampling methods described. Considering the shape of the histogram, explain which method, Method 1 or Method 2, was most likely used to obtain a such a sample.



(C) Which of the two sampling methods, Method 1 or Method 2, will result in less variability in the diameters of the 200 tortillas in the sample on a given day? Explain.

Each day, the distribution of the 200,000 tortillas made that day has mean diameter 6 inches with standard deviation 0.11 inch.

(D) For samples of size 200 taken from one day's production, describe the sampling distribution of the sample mean diameter for samples that are obtained using Method 1.

(E) Suppose that one of the two sampling methods will be selected and used every day for one year (365 days). The sample mean of the 200 diameters will be recorded each day. Which of the two methods will result in less variability in the distribution of the 365 sample means? Explain.

(F) A government inspector will visit the facility on June 22 to observe the sampling and to determine if the factory is in compliance with the advertised mean diameter of 6 inches. The manager knows that, with both sampling methods, the sample mean is an unbiased estimator of the population mean. However, the manager is unsure which method is more likely to produce a sample mean that is close to 6 inches on the day of sampling. Based on your previous answers, which of the two sampling methods, Method 1 or Method 2, is more likely to produce a sample mean close to 6 inches? Explain.

Solutions:

(A): No, a sample obtained using Method 2 will not be representative of all tortillas made that day. The sample obtained using Method 2 will only represent the tortillas from one production line, not from the entire population because the distributions of diameters for the two production lines are different.

(B): Method 1 was most likely used to select this sample. The bimodal shape in the histogram of sample data indicates that tortillas were selected from both production lines, which is what would happen using Method 1. Method 2 would be likely to produce a unimodal distribution of diameters

centered at either 5.9 inches or 6.1 inches.

(C): Method 2 would result in less variability in the sample of 200 tortillas on a given day because the sample comes from only one production line. Because the distributions of diameters are not the same for the two production lines, selecting tortillas from both lines as in Method 1 would result in more variable sample data.

(D): The sampling distribution of the sample mean diameter for samples obtained using Method 1 would be approximately normal with mean 6 inches and standard deviation $\frac{0.11}{\sqrt{200}} \approx 0.0078$ inch.

(E) Method 1 would result in less variability in the sample means over the 365 days, because with Method 2, roughly half of the sample means will be around 5.9 inches and the other half will be around 6.1 inches. With Method 1, however, the sample means will all be very close to 6 inches, as indicated by the standard deviation in part (d).

(F) Method 1 is more likely to produce a sample mean close to 6 inches. Because the sample mean is an unbiased estimator for both methods, the manager should pick the method that would result in less variability in the distribution of the sample mean. Based on the answer to part (e), Method 1 results in less variability in the distribution of the sample mean.

Problems adapted from the College Board released tests.